Effect of aging on network structure

Han Zhu,[1] Xinran Wang,[1] and Jian-Yang Zhu[2,3,*]
[1]*Department of Physics, Nanjing University, Nanjing, 210093, China*
[2]*CCAST (World Laboratory), Box 8730, Beijing 100080, China*
[3]*Department of Physics, Beijing Normal University, Beijing 100875, China*
(Received 25 April 2003; revised manuscript received 22 July 2003; published 24 November 2003)

In network evolution, the effect of aging is universal: in scientific collaboration network, scientists have a finite time span of being active; in movie actors network, once popular stars are retiring from stage; devices on the Internet may become outmoded with techniques developing so rapidly. Here we find in citation networks that this effect can be represented by an exponential decay factor, $e^{-\beta\tau}$, where τ is the node age, while other evolving networks (the Internet, for instance) may have different types of aging, for example, a power-law decay factor, which is also studied and compared. It has been found that as soon as such a factor is introduced to the Barabasi-Albert scale-free model, the network will be significantly transformed. The network will be clustered even with infinitely large size, and the clustering coefficient varies greatly with the intensity of the aging effect, i.e., it increases linearly with β for small values of β and decays exponentially for large values of β. At the same time, the aging effect may also result in a hierarchical structure and a disassortative degree-degree correlation. Generally the aging effect will increase the average distance between nodes, but the result depends on the type of the decay factor. The network appears like a one-dimensional chain when exponential decay is chosen, but with power-law decay, a transformation process is observed, i.e., from a small-world network to a hypercubic lattice, and to a one-dimensional chain finally. The disparities observed for different choices of the decay factor, in clustering, average node distance, and probably other aspects not yet identified, are believed to bear significant meaning on empirical data acquisition.

DOI: 10.1103/PhysRevE.68.056121 PACS number(s): 89.75.Fb, 89.75.Hc

I. INTRODUCTION

Recently, the computer-aided data acquisition has led to an explosion of interest in probing the complex network systems [1–3]. So far, many of the various networks in reality, such as the World Wide Web [4], social networks [5], and biological networks [6], etc., are believed to share the following characteristics [1–3,7,8]: (1) a small, relative to their large size, average distance between nodes; (2) a power-law degree distribution, often followed by a truncation; and a (3) highly clustered, (4) hierarchical, and (5) correlated structure (explanations see below). Besides, there are many other features that are often not as readily apparent and much less well understood consequently.

Aimed at a theoretical description of these findings, the Watts-Strogatz small-world model [9] is useful for systems that are largely regular, and presents properties (1) and (3) listed above. Meanwhile the Barabasi-Albert (BA) scale-free model [10] satisfactorily characterizes most of the networks where geological distance is not so important. It considers growth and preferential attachment irrespective of distance, with properties (1) and (2) as outcome. We may notice that, in this model, properties (3) and (4) are still missing [7], and, as is shown below, a neutral degree-degree correlation is predicted for large degrees (the degree of a node is defined as the number of the its links). This may be due to the highly simplified assumptions of the model. To develop these two well-established models, theoretical effort has just begun.

In the generalization of the BA model, many novel and realistic aspects have been investigated in the past few years [1,3]. It is now known that degree distribution in reality may deviate from a pure power law. According to the extent of the deviation, the distribution patterns may be categorized into three groups [11]: scale-free (power law), broad scale (power law with a sharp cutoff), and single scale (fast decaying). In order to explain this deviation, the effect of aging and a consequent loss of activity have been introduced [11–14]. This is a common mechanism in reality[1] and, combined with the BA model, it results in a tunable truncation of the power law [11,12], i.e., the degree distribution can be turned from scale-free with no aging to broad-scale with slow aging and the single-scale with fast aging. This finding justifies further investigation of its influence on the structure and function of networks, which is the aim of the present work.

This paper is organized as follows. In Sec. II, we provide empirical evidence and quantify the aging effect in a model, which reduces to the BA network when the aging effect vanishes. In Sec. III, the effect of aging on network structure is described in four aspects: clustering (Sec. III A), hierarchical

*Author to whom correspondence should be addressed. Present address: Department of Physics, Beijing Normal University, Beijing 100875, China. Email address: zhujy@bnu.edu.cn

[1]This effect can be, for example, pictured for the network of actors (the actors are linked if they both appear in the cast of one film). The more famous an actor is, the more chances he will have to act in new movies. But, however famous he may be, every star will become gradually inactive as time passes. This is also supported by the citation rate data of the years 1987–1998 [14,15], as shown in Fig. 1. Except for the first three years prior to the publication year, the citation rate gradually decreases with age.

structure (Sec. III B), degree correlation (Sec. III C), and the average distance between nodes (Sec. III D). As the intensity of the aging effect grows, in most cases we witness a continuous transformation of the network structure, while there are also a few abrupt changes. Finally, Sec. IV is the summary with some discussions.

II. THE MODEL

As the first step, we give the definition of the model with a tunable effect of gradual aging. At each time step, a newly added node is attached to m existing nodes, with the probability proportional (1) to the degree k of the considered node, as in the BA model and (2) to a simple function $f(\tau)$, where τ is the age of the considered node. Thus the evolution of the network can be approximately characterized by the following equation:

$$\frac{\partial k(t_i,T)}{\partial T}=\frac{mk(t_i,T)f(T-t_i)}{\sum_t k(t,T)f(T-t)}, \tag{1}$$

where $k(t_i,T)$ denotes the expected degree at time T of the node born at t_i, and $\Sigma_t k(t,T)f(T-t)$ is the normalization factor.

If $f(\tau)=1$, the probability that an existing node receives new links becomes solely proportional to its degree, and this model reduces to the BA model [10],

$$\frac{\partial k(t_i,T)}{\partial T}=\frac{mk(t_i,T)_i}{\sum_t k(t,T)}=\frac{k(t_i,T)}{2T}, \tag{2}$$

where we have used the fact that the normalization factor $\Sigma_t k(t,T)=2mT$. On the other hand, when $f(\tau)$ decays fast enough, we assume that the normalization factor $\Sigma_t k(t,T)f(T-t)$ reaches an asymptotic nondivergent value in the limit of infinite network size. Thus, the expected degree of a given node grows as

$$\frac{\partial k(t_i,T)}{\partial T}=\frac{k(t_i,T)f(T-t_i)}{M}, \tag{3}$$

where $M=(1/m)\Sigma_t k(t,T)f(T-t)$. To test its validity and obtain the factor $f(\tau)$, we apply this model to the scientific citation web [14–18], a rather complex network formed by the citation patterns of scientific publications, with the nodes standing for published articles and a directed edge representing a reference to a previously published paper.

Figure 1 shows the citation rate data of the years 1987–1998 [14,15] obtained from the ISI database. The number of papers published in each year [Fig. 1(a)] is approximately stable (as we have assumed), and, in order to further get rid of the fluctuation, we use a set of relative values, $F_{1987\rightarrow 1998}$, $F_{1988\rightarrow 1998}$, . . . , $F_{1998\rightarrow 1998}$, i.e., of the papers published in 1987,1988, . . . ,1998, the fraction that is cited in 1998. Then we reinterpret them as the following: each time step corresponds to a year, and as soon as a node is introduced in the Yth step, its initial degree is taken as $F_{1998\rightarrow 1998}$; within step

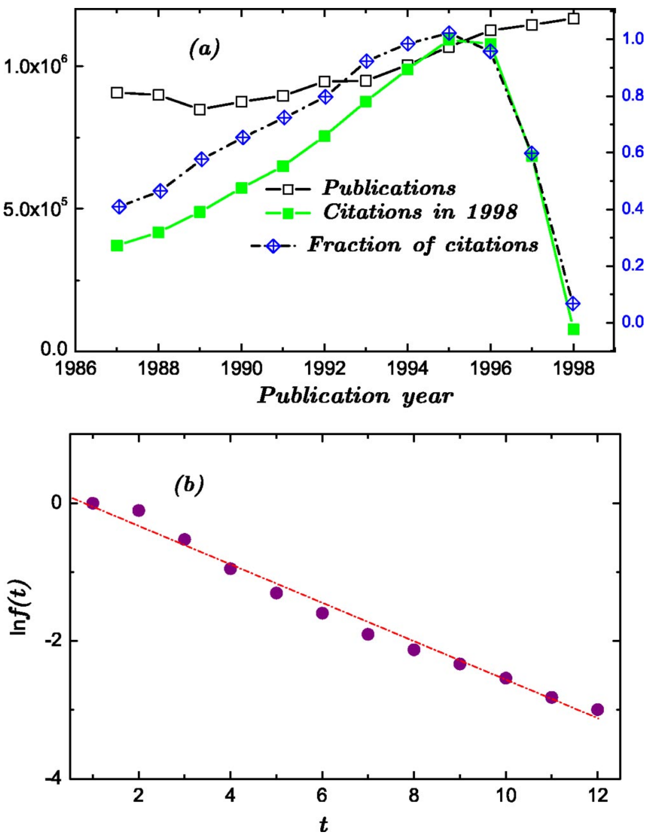

FIG. 1. (Color online) (a) The scientific citation web formed by papers (nodes) and citations (directed bonds). The open squares correspond to papers published in each year between 1987 and 1998 and the solid squares correspond to citations made in 1998 and referring to papers published in a given year [15]. The data have been extracted from the ISI database [16]. The average number of citations in a paper published in a given year received in 1998, which is actually a ratio of the other two curves, is shown in diamonds. (b) The natural logarithm of the calculated function $f(t)$ $=[1/k(t)]dk(t)/dt$ versus time step t (solid circles). The linear fit (solid line) corresponds to an exponential decay $e^{-\beta t}$, with the exponent $\beta=0.283\,53$.

$(Y+1)$, the degree increases by $F_{1997\rightarrow 1998}$; . . . within step $(Y+11)$, it increases by $F_{1987\rightarrow 1998}$, to finally $\Sigma_{i=1987}^{1998}F_{i\rightarrow 1998}$. Thus we can approximately obtain both $dk(t)/dt$ and $k(t)$ as a function of time. Then $f(t)$ can be calculated as $[1/k(t)]dk(t)/dt$, and it is shown to be proportional to, approximately, $e^{-0.28t}$ [Fig. 1(b)]. In the following we shall choose the factor to be

$$f(\tau)=e^{-\beta\tau}, \tag{4}$$

where β is a tunable parameter. This particular measurement does not exclude other functions, such as $\tau^{-\nu}$ [12,13], as possible choices. They are also studied, and found to yield similar results in most respects (while several interesting differences are highlighted below).

In the following we show how structural properties can be changed by the aging effect introduced in the way quantified above.

III. TRANSFORMATION OF NETWORK STRUCTURE

In this section, we study the transformation of network structure by the aging effect. First of all we pay attention to the mostly studied property of complex networks, the vertex degree distribution. It is well known that the aging effect may result in a transformation of the degree statistics, from scale-free with no aging, to broad scale with slow aging and to single-scale with fast aging [1,3,11,12]. The detailed analytical and numerical study of evolving networks with power-law aging that supports this idea can be found in Ref. [12]. Here, with exponential aging, we have also found by numerical simulations a very similar process.

It may be interesting to turn to the empirical results of the citation web. In Ref. [17], the network formed by papers citing each other in Physical Review D has been studied and the degree distribution significantly deviates from a power law in the range of relatively small degrees. In Ref. [18], the study has been extended to the out-degree distributions of the networks formed by papers in a variety of journals. The distributions have a maximum at intermediate out degrees, followed by an exponential tail for large out degrees (single scale). These pictures actually can be reproduced by tuning the decay factor β from small to large. The similar effect can be found in some figures of Refs. [11,12].

In the following, we report the investigation of network structure transformation by aging effect in four aspects: clustering, hierarchical structure, degree correlation, and average node distance.[2] The definitions and a brief review of relevant results can be found in the head of each section.

A. Clustering

A useful tool to characterize the network structure is the clustering coefficient C, which is defined as the average probability that a pair of nearest neighbors of a given node is also connected. For example, if the node i has k_i links, and among its k_i nearest neighbors there are ε_i edges, then the clustering coefficient of the node i is defined by

$$C_i=\frac{2\varepsilon_i}{k_i(k_i+1)}.$$

The clustering coefficient of the whole network is given by the average value. A common property of, for example, social networks is that cliques form, i.e., friends of yours are much more likely to be friends of each other than people selected at random, thus resulting in a high clustering coefficient. In the BA model, with the highly simplified assumptions, C decreases with system size N as $(\ln N)^2/N$ [20]. It is significantly lower than actual measurement, which is to a high degree unaffected by the system size [1].

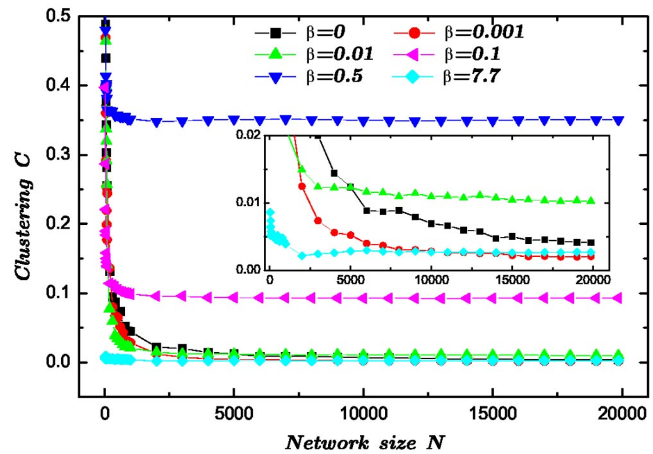

FIG. 2. (Color online) The clustering coefficient as a function of system size N, in the model described in text, with $\beta=0$ (squares), 10^{-3} (circles), 10^{-2} (upward triangles), 0.1 (leftward triangles), 0.5 (downward triangles), and 7.7 (diamonds). Each newly added node is linked to three existing nodes (also in all the following simulations). The data points correspond to system sizes varying from 100 to 20 000, and each is obtained as an average of many independent runs.

Our study of the aging effect on network structure begins with the investigation of clustering. Figure 2 shows several typical curves of the clustering coefficient $C(N)$ as a function of network size N. As aging gradually grows, we observe an interesting transformation process, which can be roughly separated into several stages. (1) $\beta=0$: The network reduces to the BA model and C decreases to finally 0 as the system grows. (2) $0<\beta<10^{-3}$: Now a slight aging effect is introduced, and C is significantly lowered given a relatively small system size, but, when the size grows, this gap is contracting. (3) $10^{-3}<\beta<10^{-1}$: As aging is becoming more and more manifest, C is greatly enhanced. Given a relatively small system size, it may still be lower than the value obtained in a nonaged network of the same size. However, with a much lower rate of decreasing, it quickly exceeds that nonaged value after reaching a crosspoint. As the system grows larger, the curve is becoming increasingly flat and C finally approaches a stable value. (4) As β continues to grow, this asymptotic value is quickly rising, as shown by the curve at $\beta=0.5$. After a certain peak is reached, it quickly falls back to finally zero.

In the following we analyze our observations. In networks, clustering is determined by a competition of orderliness and randomness. In the BA model, newly added nodes are more likely to be linked to earlier introduced nodes, which generally have more links. This orderliness, however, is weakened by the increasing randomness as the network size grows, thus resulting in a vanishing C. When the aging effect is considered, the old nodes gradually lose their activity in network function and growth. For a newly added node, it is more probable to be linked with a temporally closer node, thus forming a chainlike structure on the large scale. The region which a given node may be linked with is of a

[2] On completion of this work, we have noticed that in Ref. [19], Vazquez *et al.* have investigated the deactivation model in similar aspects. Actually, the deactivation model, just like the present model, can be viewed as a specific kind of aging effect. However, here we study gradual aging and in most cases a gradual transformation process is demonstrated by tuning the parameter. At the same time, there are some important different results, e.g., those concerning the small-world effect.

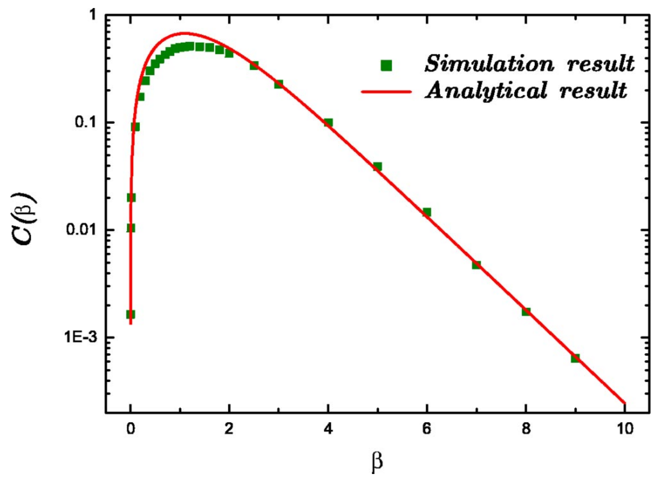

FIG. 3. (Color online) The analytical result of the asymptotic clustering coefficient of an infinitely large network, $C(\beta)$, as a function of β (line), in comparison with the simulation (squares) results. Here $m=3$ and the network size $N=20\,000$ in the simulation.

finite effective size, however large the whole network may actually be. This may explain why the $C(N)$ curve quickly reaches a stable value when the aging effect is considered. Different from the prediction of the BA model, this stable value is finite even with a vanishingly small β. Now we explain how and why this value varies with the intensity of the aging effect. In an aged network, while the global randomness as a result of the large network size is absent, the orderliness that older nodes receive more links is also lacked. The clustering coefficient of a given node is now determined by the structure of its neighboring region. When the aging effect is very weak, the size of this region is relatively large and there is an even distribution of the probability that the considered node is connected with a given member of this region. The randomness caused by such an even distribution results in a small value of C. As the aging effect grows, the size of this region contracts and the probability distribution becomes more concentrated. Thus the randomness is inhibited, and C is significantly enhanced. However, with very strong aging effect, C diminishes as the probability distribution becomes increasingly centralized. Finally, C approaches zero when each node can be linked only with the node introduced right before it.

The analysis above can be quantified by an approximate calculation of the asymptotic clustering coefficient $C(\beta)$ as a function of β. The details can be found in the Appendix and the result

$$C(\beta)=\frac{6m^3}{2m(2m-1)}e^{-\beta}\frac{(1-e^{-\beta})^3}{(1-e^{-2\beta})^2}, \tag{5}$$

in comparison with the simulation, is shown in Fig. 3. From Eq. (5) we can see that $C(\beta)$ increases linearly with β for small values of β, and decays as $e^{-\beta}$ for large values of β.

When we compare the two decay factors $e^{-\beta\tau}$ and $\tau^{-\nu}$, there is an interesting observation. In a network of 10 000 nodes, we measure the average clustering coefficient C_1 of the first half and C_2 of the second half (each containing 5000 nodes), respectively. When $e^{-\beta\tau}$ is chosen, two sets of approximately equal values are obtained; however, when $\tau^{-\nu}$ is chosen, C_2 equals C_1 only with strong or weak aging, and in the middle ground C_2 is significantly lower (Fig. 4). This interesting disparity justifies further investigation and measurement in the study of real networks.

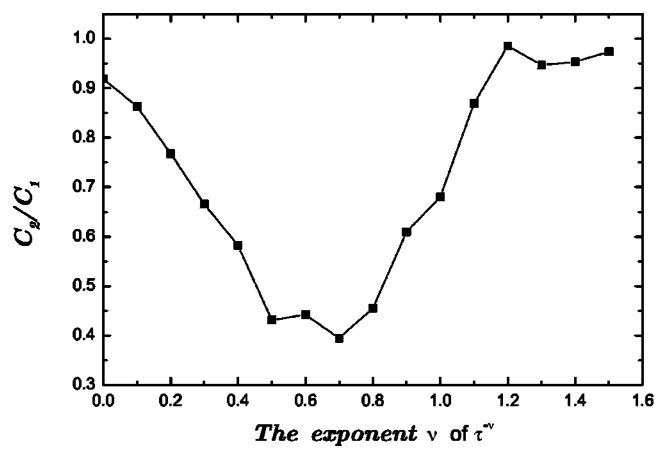

FIG. 4. In a 10 000-node network with the decay factor chosen to be $\tau^{-\nu}$, the ratio of the average clustering coefficient C_2 of the second half and C_1 of the first half, as a function of ν.

This finding also reminds us that, on empirical data acquisition, the effect of aging plays a crucial role, since researchers often have a limited access to the whole system, and will probably consider the most recently grown part. Whether such a limited investigation can correctly represent the overall system may strongly depend on the type of the aging effect.

B. Hierarchical structure

Now we go beyond the average clustering coefficient and calculate $C(k)$ as a function of k. Here $C(k)$ denotes the expected clustering coefficient of a node with k degrees.[3] For complex networks, this relationship is often of much significance because it is a useful tool to inspect the intrinsic hierarchy of the topology. In the following we briefly discuss the physical ground and then present our results with the aging effect.

In reality, networks are often fundamentally modular [6,7]: nodes have a tendency to combine into subgroups in which they are highly interconnected but have relatively few links to nodes outside. For example, in society such groups may represent families, and in World Wide Web they can denote communities with shared interests. Numerous such groups then constitute the whole system in a hierarchical manner. In some way the network might look like a fractal

[3]The method of our simulation is as follows. In each of the many independent runs, after a network is generated, the degree k and the clustering coefficient C of each node is measured. Then we calculate the average C of the nodes that have degree k. Finally, the results are further averaged over the independent runs.

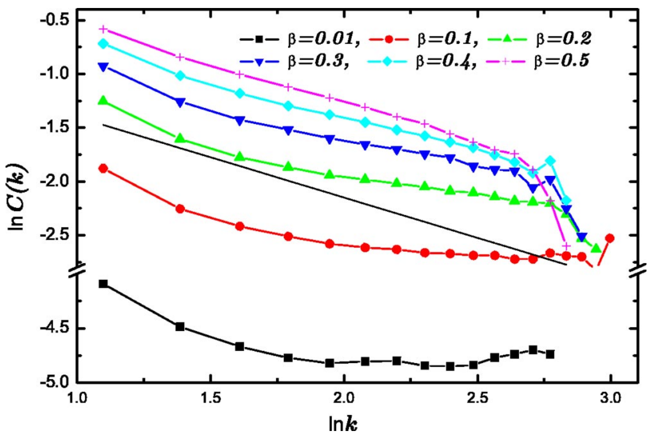

FIG. 5. (Color online) The ln-ln plot of the clustering coefficient $C(k)$ versus the connectivity k, with $\beta=0.01$ (squares), 0.1 (circles), 0.2 (upward triangles), 0.3 (downward triangles), 0.4 (diamonds), 0.5 (crosses), and the system size $N=10\,000$. The solid line is a power-law decay $k^{-\gamma}$, with the exponent $\gamma=0.75$.

graph [21] (see Fig. 1 of Ref. [7]). This structure can be characterized quantitatively by a simple scaling law: $C(k) \sim k^{-\gamma}$ [7,21]. The coefficient γ has been measured to be approximately 0.75 on the Internet at the autonomous system level [22,23].

When no aging effect is considered, the BA model does not show such a property and we expect to observe $C(k)$ as a horizontal line subject to fluctuations [7]. As aging is gradually introduced, we observe a descending slope emerging first at the leftmost part of the curve (Fig. 5). It becomes increasingly manifest until the scaling law is completely observed. The rate of the slope remains around 0.75 in the whole process. In this specific model, this value is independent of the intensity of the aging effect. This scaling law clearly indicates that a hierarchical structure is produced by the aging effect.

C. Degree correlation

In the following we discuss the aging effect on the degree-degree correlation [8] (or the mixing pattern, as it is sometimes called). For convenience we shall call a node with k degrees a $D-k$ node.

The degree correlation of nearest neighboring nodes is an important generic property of networks. It can be quantified by the probability matrix $P(k,k_{nn})$, i.e., the probability that a $D-k$ node is connected with a $D-k_{nn}$ node. However, in reality, with the available empirical data, a direct plot of $P(k,k_{nn})$ often results in a noisy picture difficult to interpret. An equivalent choice [22] is to measure instead the nearest neighbors' average degree of the $D-k$ nodes, $\langle k_{nn}\rangle_k = \sum_{k_{nn}} k_{nn} P(k,k_{nn})$, as a function of k. Following Newman's idea [8], if the high degree nodes in a network tend to connect to the low (or other high) degree nodes, then we have a disassortative (or assortative) mixing pattern; if there is no obvious bias, then we have a neutral mixing pattern and $\langle k_{nn}\rangle_k = \langle k^2\rangle / \langle k\rangle$, a value independent of k.

Before we discuss the correlation patterns with the aging effect, we provide results of the BA model for comparison. In Ref. [24], Krapivsky and Redner have obtained in the BA model a useful characterization of correlation, $N_{kl}(t)$, i.e., the number of $D-k$ nodes that attach to a $D-l$ ancestor. Asymptotically, $N_{kl}(t) \to t n_{kl}$, and

$$n_{kl} = \frac{4(l-1)}{k(k+1)(k+l)(k+l+1)(k+l+2)} + \frac{12(l-1)}{k(k+l-1)(k+l)(k+l+1)(k+l+2)}. \tag{6}$$

Here we consider undirected links and study $N'_{kl} = N_{kl} + N_{lk}$, the number of $D-k$ nodes that are *linked with* a $D-l$ node. Asymptotically,

$$N'_{kl}/t \to n'_{kl} = n_{kl} + n_{lk}.$$

The probability that a nearest neighbor of a $D-k$ node is $D-k_{nn}$ is

$$(N_{k,k_{nn}} + N_{k_{nn},k}) \Big/ \sum_{k_{nn}} (N_{k,k_{nn}} + N_{k_{nn},k}),$$

and the average degree of the nearest neighbors of the $D-k$ nodes is

$$\langle k_{nn}\rangle_k = \frac{\sum_{k_{nn}} k_{nn}(N_{k,k_{nn}} + N_{k_{nn},k})}{\sum_{k_{nn}} (N_{k,k_{nn}} + N_{k_{nn},k})} \to \frac{\sum_{k_{nn}} k_{nn}(n_{k,k_{nn}} + n_{k_{nn},k})}{\sum_{k_{nn}} (n_{k,k_{nn}} + n_{k_{nn},k})}. \tag{7}$$

We show it approximately in Fig. 6(a) by taking the summation of k_{nn} to 1.5×10^5 and 2×10^6 (with normalization satisfied) respectively, in comparison with the simulation results at $N=100$, 1000, and 10 000. It is found that nodes with large k show no obvious biases in their associations. But, there is a short disassortative mixing region when k is relatively small.

Now we introduce a tunable aging effect. With the parameter β taking different values, the ln-ln plots of $\langle k_{nn}\rangle_k$ are shown in Fig. 7(a). When k is relatively small, they all descend linearly with approximately the same slope. This indicates that in this region $\langle k_{nn}\rangle_k$ decays as a power law, $k^{-\lambda}$, and the exponent λ is largely independent of the intensity of the aging effect. But they show different trends as k increases. (1) For small values of β [$\beta=0.01$ and 0.1 in Fig. 7(a)], the curves become flatter as k increases. (2) For large β [$\beta=0.4$ and 0.5 in Fig. 7(a)], the power law is truncated with a fast decaying tail. (3) In the middle ground [$\beta=0.2$ and 0.3 in Fig. 7(a)], the power law is maintained. To conclude, the aging effect, possibly above a certain intensity, leads to a disassortative mixing pattern.

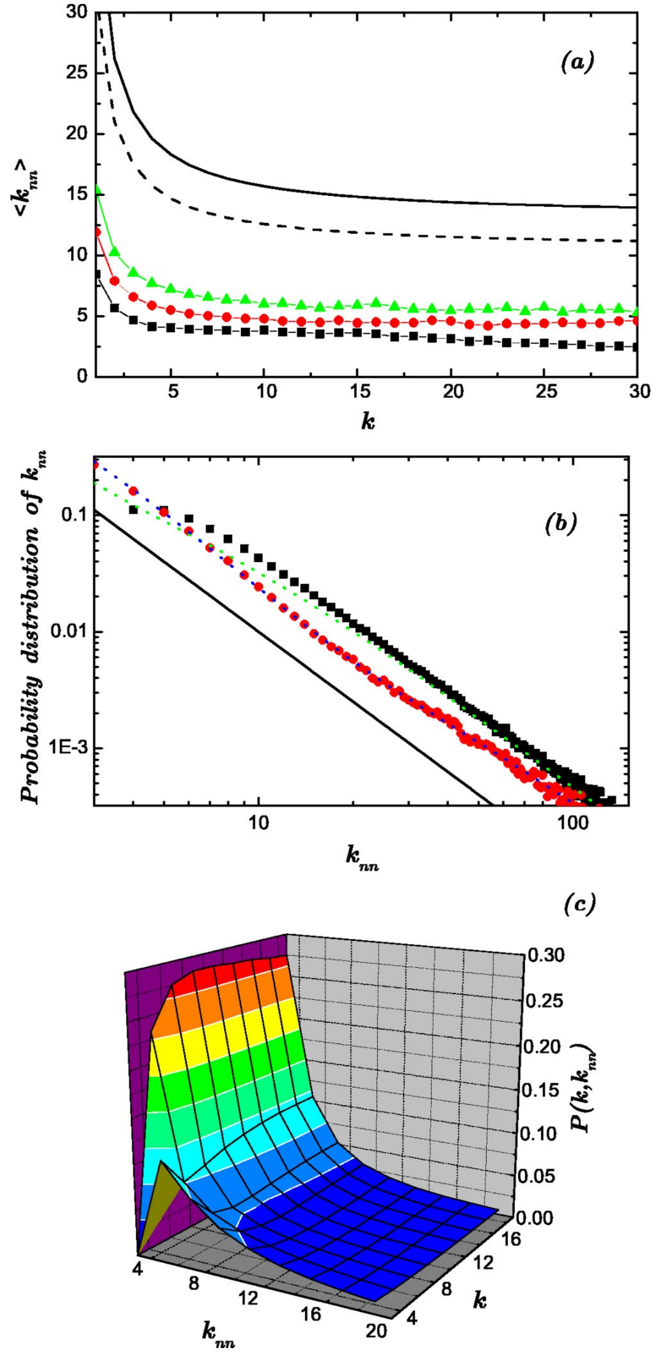

FIG. 6. (Color online) Degree-degree correlations in the BA model without aging. (a) Average degree $\langle k_{nn}\rangle$ of the neighboring nodes of the $D-k$ nodes as a function of k. There is no aging effect and $\beta=0$. Squares, circles, and upward triangles correspond to the simulation results with system size $N=100$, 1000, and 10 000, respectively. The dashed line and the solid line represent the theoretical results with k_{nn} up to 1.5×10^5 and 2×10^6, respectively. (b) Degree distributions of the nearest neighbors of $D-3$ nodes (squares) and $D-20$ nodes (circles), respectively. The dashed lines are the corresponding theoretical results. The solid line with slope -2 serves as a guide to the eye. (c) The probability matrix $P(k,k_{nn})$. In both (b) and (c), system size $N=10\,000$.

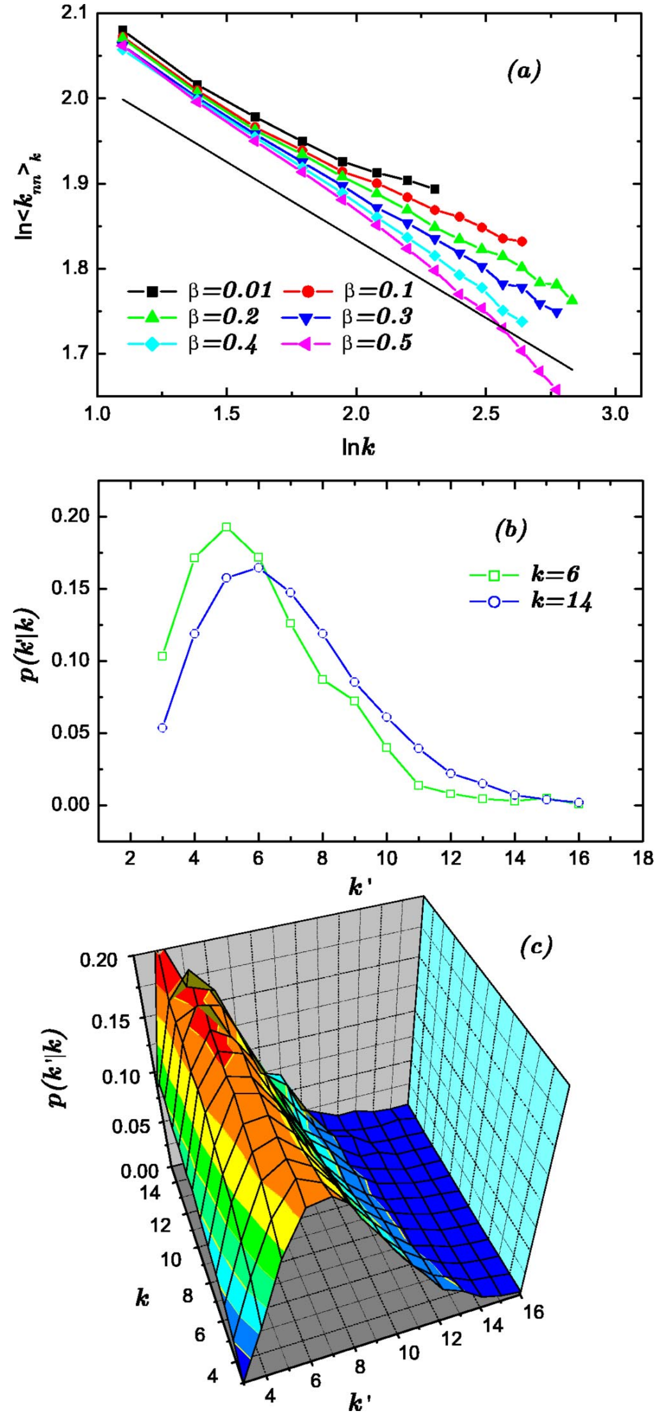

FIG. 7. (Color online) Degree-degree correlations with aging effect. (a) The ln-ln plot of $\langle k_{nn}\rangle_k$ versus k, at $\beta=0.01$ (squares), 0.1 (circles), 0.2 (upward triangles), 0.3 (downward triangles), 0.4 (diamonds), and 0.5 (leftward triangles), with system size $N=10\,000$. The solid line corresponds to a power-law decay $k^{-\lambda}$, with the exponent $\lambda=0.183$. (b) Degree distributions of the nearest neighbors of $D-6$ nodes (circles) and $D-14$ nodes (squares), respectively. (c) The probability matrix $P(k,k_{nn})$. Both (b) and (c) are obtained with $N=3000$.

To explain the emergence of this mixing pattern, we directly study the probability matrix $P(k,k_{nn})$. In the BA model, Fig. 6(b) shows two characteristic distributions of the probability that a nearest neighbor of a $D-k$ node is $D-k_{nn}$. They are shown to be declining as approximately k_{nn}^{-2} for large k_{nn}, which is a sign of neutral mixing since the probability of finding a $D-k_{nn}$ node in the network is approximately k_{nn}^{-3} [10,24]. It also leads to the conclusion that $\langle k_{nn}\rangle$ will diverge for infinite network size N as $\ln N$, since the largest possible value of $k\sim N^{1/2}$ [10], and approximately

$$\langle k_{nn}\rangle \sim \frac{\int_1^{\sqrt{N}} k_{nn}k_{nn}^{-2}\, dk_{nn}}{\int_1^{\sqrt{N}} k_{nn}^{-2}\, dk_{nn}} \sim \ln N. \tag{8}$$

This trend is observed in Fig. 6(a). A three-dimensional drawing of the matrix $P(k,k_{nn})$ without aging can be found in Fig. 6(c). When aging is considered, the effect is remarkable, as is shown in Figs. 7(b) and 7(c). On the one hand, the probability distribution is changed from a power law to a bell-shaped type [Fig. 7(b)]; on the other hand, with k growing, this bell-shaped distribution is shifting translationally, along the negative direction of the axis of k_{nn}.

D. Average distance between nodes

Finally, we study another fundamental topological feature of complex networks, the average node-node distance $\langle d\rangle$. Here the distance between two selected nodes is defined as the number of edges along the shortest path connecting them [1–3]. In our simulation we calculate $\langle d\rangle$ as $\Sigma_{i=1}^{N}\langle d_i\rangle/N$, where $\langle d_i\rangle$ is the average distance between the node i and the rest of the network.

As is mentioned in the Introduction, many complex networks show striking small-world properties and have a relatively small value of $\langle d\rangle$ compared with their large size. This effect is shared by many models, including the small-world model, the BA model and the random network. However, with the aging effect strong enough, it is imaginable that each node could only be connected with those introduced shortly before it. Thus we may probably have a chainlike structure, with $\langle d\rangle$ scaling linearly as N. In fact, as is shown below, this depends on the choice of the decay factor.

(1) When $e^{-\beta\tau}$ is applied, we obtain a chainlike structure even with vanishingly small β [β=0.01 in Fig. 8(a)], and $\langle d\rangle$ increases linearly as N. However, a chainlike structure does not necessarily mean a large value of $\langle d\rangle$. Here $\langle d\rangle$ is still relatively small compared with N. (2) When $\tau^{-\nu}$ is applied, we observe a continuous transformation process that can be roughly identified with three stages: The first stage is a small-world network with $\langle d\rangle\sim\ln N$ [ν=1 in Fig. 8(b)]; in the second stage the network is similar to a hypercubic lattice with $\langle d\rangle\sim N^{1/D}$, where D is the Euclidean dimension [ν=2 and $D\approx$2.73 in Fig. 8(c)]; finally, in the third stage, the network evolves into a chainlike structure with $\langle d\rangle\sim N$ [Fig. 8(d)]. The difference between the two decay factors becomes more manifest when we take into consideration the position

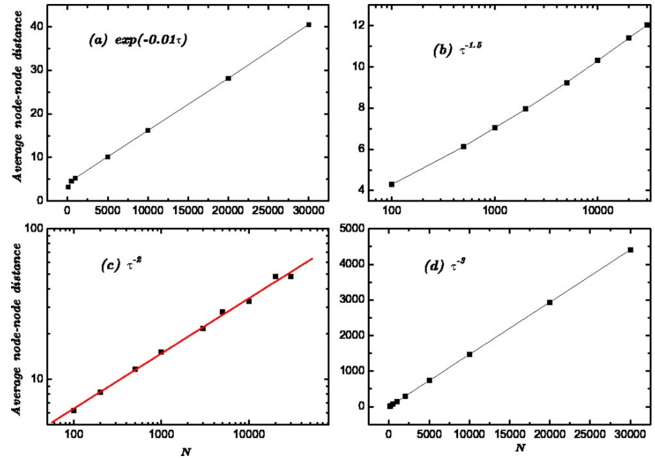

FIG. 8. (Color online) Average distance between nodes as a function of network size N with different decay factors: (a) $f(\tau)=e^{-0.01\tau}$ and $\langle d\rangle\sim N$, (b) $f(\tau)=\tau^{-1.5}$ and $\langle d\rangle\sim\ln N$, (c) $f(\tau)=\tau^{-2}$ and $\langle d\rangle\sim N^{-1/2.73}$, and (c) $f(\tau)=\tau^{-3}$ and $\langle d\rangle\sim N$. The results are obtained with $m=2$.

of each node and plot $\langle d_i\rangle$ as a function of i. Figure 9(a), obtained with the decay factor $e^{-0.01\tau}$, is just a sign of a chainlike structure. By contrast, in Fig. 9(b) with the decay factor τ^{-1}, the nodes born earlier have shorter distance.

For each node, the probability that it is linked with a newly introduced node is proportional to its degree and the decay factor, and the structure of the generated network is decided by such a competition. A major difference between the two decay factors, $e^{-\beta\tau}$ and $\tau^{-\nu}$, is that $e^{-\beta\tau}$ decays much faster. With $e^{-\beta\tau}$, the probability that two temporally distant nodes are connected is so small that a chain-like structure will be produced even with vanishingly small β. However, when $\tau^{-\nu}$ is chosen, the result of the competition depends on the parameter ν, and that is why we have observed a continuous transformation. Besides, it is worth mentioning that actually the small-world property can be retained with ν in a remarkably large region.

Finally, it is worth mentioning that, from the observation of an exponential aging in the citation network and the present model, we may predict a linear increase of the average distance between nodes (in the citation web) with the

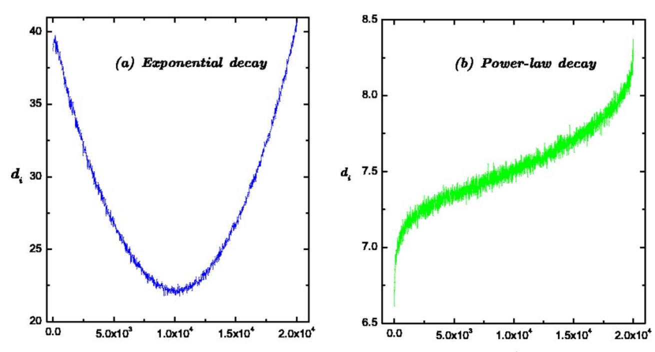

FIG. 9. (Color online) The average distance between each given node in the network and all the other nodes, with the decay factor as (a) $f(\tau)=e^{-0.01\tau}$ and (b) $f(\tau)=\tau^{-1}$. Here $N=20\,000$ and $m=2$.

number of nodes in the network. This is against the common expectation of a ''small-world behavior,'' where a logarithmic increase is observed, but it is certainly what follows from our work. We hope this conclusion will stimulate further statistical measurements of citation networks.

IV. SUMMARY

In network evolution, aging is a universal effect: After all, nothing is perpetual. Here we find in citation networks that the aging effect may be represented by an exponential decay factor, $e^{-\beta\tau}$, while this particular measurement does not exclude the possibility that other evolving networks may have different types of aging, for example, a power-law decay, which is also studied and compared. It has been found that, as soon as the preferential attachment is modified by such a factor, the produced network will be significantly transformed, besides the change of the degree distribution [11,12]. In most cases we observe a continuous transformation process by tuning the decay factor, while there are also a few abrupt changes. (1) The network will be clustered even with infinite network size, and the clustering coefficient varies greatly with the intensity of the aging effect, i.e., it increases with β linearly for small values of β and decays exponentially for large values of β. At the same time, the aging effect may also result in (2) a hierarchical structure and (3) a disassortative degree-degree correlation, and we observe how the corresponding scaling laws gradually emerge. (4) Generally the aging effect will increase the average node-node distance in a network, but the result depends on the chosen decay factor and the intensity: If exponential decay is applied, the network appears like a one-dimensional chain; but with power-law decay a transformation process is demonstrated, i.e., from a small-world, to a hypercubic lattice, and to a one-dimensional chain finally.

Presently there are plenty of problems worthy of further investigation. For example, the influence of different choices of the decay factor, hidden behind similar statistical properties, on network structure. In the present research, the interesting disparities revealed, concerning the clustering and the node distance, and probably with other aspects not yet identified, are believed to bear significant meaning for empirical data acquisition. We hope that the measurement conducted in the present work, about the citation web, will be applied to more systems in future empirical studies. The effect of aging on network structure observed in the present paper also justifies a parallel study of the effect on network function [1], which includes topics such as efficiency, information and disease transportation, error and attack tolerance, percolation features, etc.

ACKNOWLEDGMENTS

The authors would like to thank Konstantin Klemm for his help in providing the original data used in Fig. 1. This work was supported by the National Natural Science Foundation of China under Grant No. 10075025.

APPENDIX: CALCULATION OF THE ASYMPTOTIC CLUSTERING COEFFICIENT

Because of the decay factor $e^{-\beta\tau}$, each node can only be linked with a finite region, however large the whole network may actually be. We can use this property to simplify the calculation of the asymptotic clustering coefficient of a infinitely large network.

As is described in Sec. II, the model network is built as the following. At each time step a single node is introduced, and then it is attached to m existing nodes. After the network is built, we calculate the clustering coefficient of a randomly selected node, which is numbered as node 0. We number the node introduced i time steps before it as node $-i$ and the node introduced i time steps after it as node $+i$. As a result of the aging effect, we can assume that each node of this network has the same connectivity $2m$. Thus, the probability that two nodes i and j are connected can be written as (a node cannot be linked with itself)

$$P_{link}(i,j)=m\frac{2m(e^{-\beta|i-j|}-\delta_{ij})}{\sum_{l=1}^{\infty}2me^{-\beta l}} =m\frac{e^{-\beta|i-j|}-\delta_{ij}}{\sum_{l=1}^{\infty}e^{-\beta l}} =m(e^{\beta}-1)(e^{-\beta|i-j|}-\delta_{ij}). \tag{A1}$$

The clustering coefficient of the node 0,

$$C(\beta)=\frac{\sum_{i=-\infty}^{+\infty}\sum_{j=-\infty}^{+\infty}P_{link}(0,i)P_{link}(0,j)P_{link}(i,j)}{2m(2m-1)} =\frac{m^3(e^{\beta}-1)^3}{2m(2m-1)}\left[2\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}e^{-\beta i}e^{-\beta j}(e^{-\beta|i-j|}-\delta_{ij}) +2\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}e^{-\beta i}e^{-\beta j}e^{-\beta(i+j)}\right] =\frac{6m^3}{2m(2m-1)}e^{-\beta}\frac{(1-e^{-\beta})^3}{(1-e^{-2\beta})^2}.$$

In fact, this value does not depend on which node we select, and thus it can be taken as the clustering coefficient of the network. As is shown in Fig. 3, this approximate calculation has a better fit with the simulation results for large values of β. Actually, the difference mainly comes from Eq. (A1), in which we do not consider the degree difference of the nodes.

[1] R. Albert and A.-L. Barabasi, Rev. Mod. Phys. **74**, 47 (2002).
[2] S.H. Strogatz, Nature (London) **410**, 268 (2001).
[3] S.N. Dorogovtsev and J.F.F. Mendes, Adv. Phys. **51**, 1079 (2002).
[4] B.A. Huberman and L.A. Adamic, Nature (London) **401**, 131 (1999).
[5] S. Wasserman and K. Faust, *Social Network Analysis* (Cambridge University Press, Cambridge, 1994).
[6] E. Ravasz, A.L. Somera, D.A. Mongru, Z.N. Oltvai, and A.-L. Barabasi, Science **297**, 1551 (2002).
[7] E. Ravasz and A.-L. Barabasi, Phys. Rev. E **67**, 026112 (2003).
[8] M.E.J. Newman, Phys. Rev. Lett. **89**, 208701 (2002); Phys. Rev. E **67**, 026126 (2003).
[9] D.J. Watts and S.H. Strogatz, Nature (London) **393**, 440 (1998).
[10] A.-L. Barabasi and R. Albert, Science **286**, 509 (1999).
[11] L.A.N. Amaral, A. Scala, M. Barthelemy, and H.E. Stanley, Proc. Natl. Acad. Sci. U.S.A. **97**, 11 149 (2000).
[12] S.N. Dorogovtsev and J.F.F. Mendes, Phys. Rev. E **62**, 1842 (2000).
[13] S.N. Dorogovtsev and J.F.F. Mendes, Phys. Rev. E **63**, 046107 (2001).
[14] K. Klemm and V.M. Eguiluz, Phys. Rev. E **65**, 036123 (2002).
[15] A.F.J. van Raan, Scientometrics **47**, 347 (2000).
[16] See http://www.isinet.com
[17] S. Redner, Eur. Phys. J. B **4**, 131 (1998).
[18] A. Vazquez, e-print cond-mat/0105031.
[19] A. Vazquez, M. Boguna, Y. Moreno, R. Pastor-Satorras, and A. Vespignani, Phys. Rev. E **67**, 046111 (2003).
[20] K. Klemm and V.M. Eguiluz, Phys. Rev. E **65**, 057102 (2002).
[21] S.N. Dorogovtsev, A.V. Goltsev, and J.F.F. Mendes, Phys. Rev. E **65**, 066122 (2002).
[22] R. Pastor-Satorras, A. Vazquez, and A. Vespignani, Phys. Rev. Lett. **87**, 258701 (2001).
[23] A. Vazquez, R. Pastor-Satorras, and A. Vespignani, Phys. Rev. E **65**, 066130 (2002).
[24] P.L. Krapivsky and S. Redner, Phys. Rev. E **63**, 066123 (2001).